Efficient Implementation of the Cauchy Method for Automated CAD-Model Construction

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***Abstract*—An efficient and stable implementation of the multidimensional Cauchy method for creating high quality multivariate models of microwave circuits from electromagnetic (EM) simulation data is presented in this paper. The algorithm uses the total least squares method to solve the ill-conditioned interpolation problem and automatically determines the model order and distribution of support points in the parameter space in such a manner that instabilities are prevented. Numerical tests show that the method requires fewer support points to achieve similar accuracy as the rational function models published previously. The utility and accuracy of the technique is demonstrated on a filter design example involving three- and five-dimensional models.**

***Index Terms*—Adaptive multivariate rational interpolation, CAD models, Cauchy method.**

I. Introduction

SURROGATE parametrized models of microwave components constructed from data provided by a full-wave electromagnetic solver are one of the key building blocks of the successful CAD of complex microwave circuits.

In recent years, the problem of creating models based on electromagnetic simulations has received considerable attention. One of the goals of the research in this area is to create techniques that allow one to build a multidimensional parametric model of a component using as few data points as possible. An example of the technique which can be used to achieve this goal are artificial neural networks (ANN) [1]. However, the difficulties related to finding the proper topology and long training process make the ANN hard to apply in automated model generation procedures. Automatic model creation was presented in [6]. In this proprietary algorithm, frequency is handled separately from other physical parameters. At selected frequency points multidimensional models are created by expanding the multivariate functions into series of orthogonal multinomials. The expansion coefficients are found by solving a system of interpolatory conditions. Support points are added in an entirely adaptive way. Orthogonal multinomials improve the numerical stability and efficiency of the interpolation. The frequency dependence is added by one-dimensional rational interpolation of the models' response. The procedure creates models with good accuracy, but it is obvious that excluding frequency from the adaptive sampling procedure may result in nonoptimal number of support points.

Models can also be created in an automated fashion by interpolation EM data with multivariate rational functions. The most straightforward approach is to extend the univariate Cauchy method [4], which allows adaptive selection of support points [2] and model order, to higher dimensions by setting up and explicitly solving a system of interpolatory conditions. This approach involves a large ill-conditioned system of equations that is expensive to solve. Since the adaptive selection of support points and model order requires solving the system many times, the technique is considered computationally ineffective, inaccurate and suitable for simple models [5], [7]. In fact, [5] shows the results for two-dimensional models. For higher dimensions a fast and stable recursive Burlisch-Stoer algorithm was developed [5] in which the adaptive sampling can be applied only in one dimension and all other samples have to form a completely filled uniform or nonuniform grid. This implies that the number of full wave analyzes is high. To reduce the number of support points while retaining the speed and stability of interpolating algorithm Lehmensiek and Meyer [7] developed techniques based on Thiele-type branched continued fraction representation of a rational function. The algorithms operate by using a univariate adaptive sampling along a selected dimension. In this way, while the support points do not fill the grid completely, they are being added along straight lines passing through multidimensional space. The efficiency of the algorithms was illustrated on two- and three-dimensional models.

In this letter we show that despite earlier skepticism multivariate rational interpolation that involves setting up and explicitly solving a system of interpolatory condition can be implemented in such a way that accurate high dimensional models can be created automatically with support points added along all dimensions (including frequency) simultaneously. From the mathematical point of view, the starting point in our approach and the basic idea for adaptive sampling is similar to that presented in [5]. The algorithm has however been developed in such a way that it becomes efficient and stable.

II. Multivariate Cauchy Method

We start with highlighting the key points of the multivariate Cauchy method. The method interpolates a real or complex valued function of N-variables $S(\underline{X}) = S(x_1, x_2, \ldots, x_N)$ with a rational function

$$\widehat{S}(x_1, x_2, \ldots, x_N) = \frac{A(\underline{X})}{B(\underline{X})} = \frac{A(x_1, x_2, \ldots, x_N)}{B(x_1, x_2, \ldots, x_N)} \tag{1}$$

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where both numerator $A(\underline{X})$ and denominator $B(\underline{X})$ are multinomials. Multinomials are linear combinations of monomials involving product of all variables raised to different powers [8]. In the interpolation problem one looks for the values of coefficients of multinomials in numerator and denominator that ensure $\widehat{S}(\underline{X}) = S(\underline{X})$ at all support points. Let M_1 be the number of a_i coefficients associated with numerator and M_2 be the number of b_i associated with denominator of (1). Coefficients a_i and b_i can be found by requiring that

$$A(\underline{X}) - S(\underline{X})B(\underline{X}) = 0 \quad (2)$$

is fulfilled in at least $L = M_1 + M_2$ support points.

The rational function fitting problem can be written in the matrix form as

$$[A - B]\begin{bmatrix} a \\ b \end{bmatrix} = 0 \quad (3)$$

where a, b are the vectors of unknown coefficients and $[A]_{L\times M_1}$, $[B]_{L\times M_2}$ are matrices involving the values of the monomials appearing in numerator and denominator of (1) as well as the values of the interpolated function at support points. Adding new support points adds new equations to (3) and the system becomes overdetermined. The new support points are added in an adaptive way using the technique described in [2] and [5]. In this technique two interpolants of different order are constructed and compared. A new sample (and a new equation) is added at the point where the two interpolants show the biggest mismatch.

III. Implementation Details

We now pass to the implementation details which are crucial for numerical efficiency and stability of the whole procedure. Problems involving interpolation with nonorthogonal multinomials are ill-conditioned [8], therefore the method used to solve system (3) has to be particularly robust. Following [3] we have selected the total least squares (TLS) method [9]. From the point of view of model creation the TLS method has one additional advantage. The most time consuming step involves the QR decomposition of matrix $C = [A - B]$. Adding a new support point to the system appears to require a new QR decomposition. Rather than doing that we compute the full QR decomposition only once for a given interpolation model. As new points are added the QR decomposition is updated using row update procedure described in [9]. The QR update is much faster than a full QR factorization. For example, the QR factorization of a complex matrix with 1700 rows and 490 columns takes over 32 s, while QR update takes only 1 s.

Despite robustness of the TLS method, the algorithm may become unstable. To prevent this one has to monitor the interpolation error at each step. The mismatch between two interpolants serves as an estimate of the interpolation error. New support points are added, one at a time, as long as the error decreases. The sudden error increase is the symptom of poor stability of the numerical solution. Once this happens, the stability is restored by adding more than one support point. This is achieved by dividing the N-parameter space into 2^N sub-spaces. In each subspace the point of biggest interpolant mismatch is found and the locations of these 2^N maxima are added to the set of support points. This procedure is repeated until the stability is restored, and in next step the model over original set of parameters is computed. If N is small only a series of QR updates is carried out. A similar procedure is used if a cluster of support points forms.

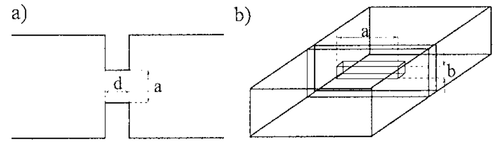

Fig. 1. Symmetric capacitive step (a) and iris (b) in rectangular waveguide.

It has to be noted that we were unable to find rigorous theory why adding more points restores stability, our explanation of this fact is as follows. The TLS method operates on the overdetermined system of equation and instability appears when a support point is ill-selected. When many points are added at the same time the TLS is able to find a solution, in which the ill-selected point does not contribute.

The interpolation error estimate, that is used to select new support points, is also used to adaptive model order selection. The procedure starts with two different low order interpolants and new support points are added using the stabilized adaptive sampling algorithm described above. This continues until the error drops below the desired level, which indicates that the model has converged, or until the stagnation is detected. Stagnation is defined as the situation where no significant error improvement is observed during successive $K = 2^N$ steps. If this takes place, the degrees of the multinomials in numerator and denominator are simultaneously increased. Increasing the model order entails adding several monomials at once to both numerator and denominator so more than one support points have to be added. Again, the parameter space is divided into a suitable number of subspaces and support points are added at places where the biggest interpolants' mismatch occurs. Everytime the model order is increased a full QR factorization is performed.

For the high complexity functions presented algorithm needs many memory resources to compute the model coefficients with TLS. Our suggestion is to limit the maximum size of the solved system and if it is exceeded to divide the parameter space into smaller subdomains, where the models can be easily evaluated.

IV. Numerical Results

To compare our algorithm with that of [7] two trivariate models for (complex) reflection and transmission coefficients of waveguide discontinuities were constructed. The mode-matching method was used to compute the EM data. The first model was created for reflection coefficient $S_{11}(f, a, d)$ of a symmetric capacitive step [Fig. 1(a)] in a standard WR90 waveguide with $f \in [7\text{ GHz}, 13\text{ GHz}]$, iris width $a \in [2\text{ mm}, 8\text{ mm}]$ and thickness $d \in [0.5\text{ mm}, 5\text{ mm}]$. The second model was determined for the transmission coefficient $S_{21}(f, a, b)$ of an iris [Fig. 1(b)], with $f \in [8\text{ GHz}, 12\text{ GHz}]$, $a \in [8\text{ mm}, 15\text{ mm}]$ and $b \in [1\text{ mm}, 3\text{ mm}]$. To evaluate the accuracy the error function $\epsilon = \|\widehat{S}(\underline{X}) - S(\underline{X})\|$ is maximized in the parameter space with a global optimization procedure.

TABLE I
EXAMPLE RESULTS

Structure	Support points	Maximum error [dB]
Step	112	−39.02
Step	301	−50.98
Iris	75	−34.1
Iris	208	−53.91

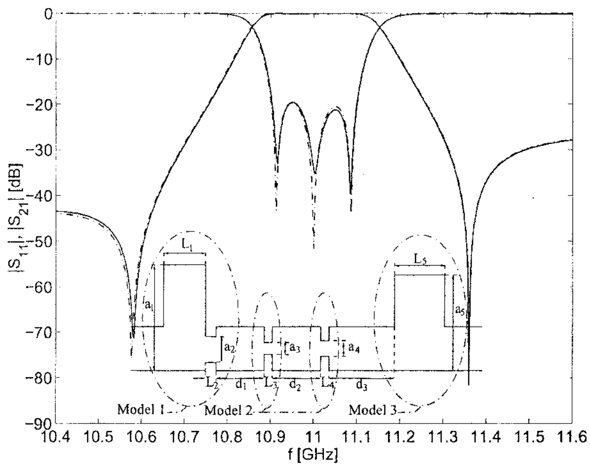

Fig. 2. Scattering parameters and structure (top-view) of the third order filter : — model, -.- mode-matching. WR75 waveguide, structure dimensions : $a_1 = 34.22$ mm, $a_2 = 12.05$ mm, $a_3 = 8.82$ mm, $a_4 = 8.4$ mm, $a_5 = 36.45$ mm, $L_1 = 20$ mm, $L_2 = 1$ mm, $L_3 = 3.34$ mm, $L_4 = 3.73$ mm, $L_5 = 16.03$ mm, $d_1 = 9.23$ mm, $d_2 = 16.2$ mm, $d_3 = 11.98$ mm.

Table I shows number of points required to create models and corresponding accuracy of the models relative to the mode-matching computations. The error was defined as $20\log(\epsilon_{\max})$.

It can be seen that we need only 301 (step) or 208 (iris) support points to achieve the accuracy below -50 dB, which is sufficient in most applications. Let us note that this is significantly fewer support points than what was required with the method presented in [7] (from approximately one fifth, for the step to one fourth, for the iris).

A. Filter Design

To verify the quality of models generated by the proposed technique and demonstrate that even very complex models can be created, the third order waveguide filter shown in Fig. 2 was decomposed into discontinuities. Then three discontinuity models of 5, 3, and 3 variables were generated with accuracy better than -40 dB and connected by waveguide sections. For example, in the 5-variate case (the discontinuity denoted as Model 1 in Fig. 2), all S_{11}, S_{21} and S_{22} parameters were modeled with frequency range $f \in [10.5\ \text{GHz}, 11.5\ \text{GHz}]$, and other parameters : $a_1 \in [14\ \text{mm}, 20\ \text{mm}]$, $L_1 \in [32\ \text{mm}, 45\ \text{mm}]$, $a_2 \in [3\ \text{mm}, 6.5\ \text{mm}]$ and $L_2 \in [1\ \text{mm}, 5\ \text{mm}]$. To generate a models of such complexity two different sets of fewer than 1700 support points were used. Fig. 2 compares the scattering parameters of the filter computed using the models with the full-wave mode-matching simulations. Good agreement is seen, which indicates that our implementation of the multidimensional Cauchy method yields high quality complex models which may be used in CAD of microwave circuit.

V. CONCLUSIONS

A new implementation of the algorithm proposed in [5] for automated model creation with multidimensional adaptive Cauchy method was described. Stability and efficiency were achieved by using TLS method for solving the interpolation equations combined with the QR update and the monitoring of interpolation error to detect convergence or potential instability and automatically select support points and model order. Clustering of support points and instability are prevented by parameter space division.

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